



# REINSURANCE WITH R: PRICING, RESERVING, SOLVENCY AND PORTFOLIO ANALYSIS

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## ABSTRACT

The lack of education and training has been highlighted as one of the highest factors stifling the growth of the re/insurance industry in Africa. Although reinsurance is offered as a course in the insurance/actuarial undergraduate and postgraduate study programs, many higher institutions of learning in Nigeria lack the sufficient mathematical and technical faculty expertise required to thoroughly deliver the reinsurance subject matter particularly at the masters and doctorate levels. As a result of the general absence of the required expertise in the re/insurance field of study, only a couple of universities offer postgraduate programs in insurance within Nigeria. The ripple effect of this gap is consequently felt in the re/insurance sector. This paper, therefore, is meant to serve as a teaching aid to boost the postgraduate teaching and classroom experience in reinsurance and to strengthen the technical capacity of researchers in this field. First, contributions to literature from the Nigerian academic landscape is reviewed and then the computational aspect is implemented. The study aims to bridge the gap between theory and application. Both simulated and real data are used to illustrate the different concepts in reinsurance with techniques ranging from Monte Carlo simulations to various statistical distributions. The R language has been adopted because of its ease of use.

**Keywords:** *Aggregate claims; Reinsurer's risk; Pareto distribution; Frequency-severity; Nigerian reinsurance sector.*

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## INTRODUCTION

In Nigeria, only a handful of higher learning institutions offer insurance/actuarial science as an undergraduate degree program. Fewer still, offer it at the master's degree level with actuarial science rarely making an appearance. Reinsurance stands out as one of the courses that need to be studied and deeply understood at the postgraduate level. Its content is quite wide covering aspects relating to the different classes of reinsurance business and other aspects of reinsurance as it pertains to pricing, accounting, the structural design/construction, loss reserving methods, the placement process, portfolio management, reinsurance legal frameworks and market operations. Sadly, a thorough grasp of this course is quite elusive within Nigerian institutions despite the many benefits that reinsurance offers. What complicates this issue the more lies firstly, in the detailed mathematical structure that frames the reinsurance course, its complex environment combines both capital and catastrophe models, different ratings as well as treaty forms which draw heavily from mathematics and statistics. Secondly, the huge re/insurance databases providing large amounts of information on different risks require the use of computer programs (guided by mathematical logic) to investigate the behaviour of the various risky policies. Thirdly, and more disheartening is the lack of experts/researchers with a special interest in this field of study. It then, should not come as a surprise that the sector which is a knowledge-based industry (Swiss Re, 2019a ) is seriously lagging.

It is important to also emphasize that the short supply of reinsurance specialists is not unique to Nigeria. Globally, this is the case as well (Carl, 2021). With the advent of blockchain digital techniques, Calandro et al., (2018) report that routine administrative tasks such as accounting, renewal tasks, general reporting and contract supervision will easily be accomplished with technology, however complex tasks that involve comprehensive analytical processes, in particular, challenging claim types and underwriting will have to be executed by trained specialists and qualified staff.

Schanz (2019) further stressed the dire shortage of skilled labour within Africa, stating that experienced talent is highly needed for the advancement of risk management, the development of innovative products and technologies as well as the optimal use of market opportunities. The author also voiced out the non-existing capacity for some highly specialized risks in some African markets and the consequent over-reliance on international reinsurers which tend to exclude the local players. With the rising challenges stemming from the changing climate, Africa can no longer afford to keep dragging her foot with regard to the training and upskilling her own. Africa is the hardest hit continent by climate change given that more than half of its citizens rely on agriculture. This further implies a greater need for re/insurance.

In 2019, the 24th African reinsurance forum, one of the major events of the African Insurance Organization (AIO), was held in Tunisia (Allianz, 2019). The focus was on the new regulatory challenges facing the African re/insurance industry. Although commendable strides have been made, of note is the inclusion of over 21 African members (who represent 35 countries) in the International Association of Insurance Supervisors (IAIS), the challenges still abound, particularly in the aspect of the absence of both the financial capacity and skills needed to re/insure specialist risks (Traore, 2019). This fully aligns with other earlier reports. For instance, Trust Re (2014) released a summary report on the outlook for insurance and reinsurance in sub-Saharan Africa. The participants included 151 directors,

executives, top managers and other insurance professionals pooled from 32 African countries. A sore point that was continuously highlighted was the lack of capacity for dealing with big complex risks and the limited availability of both technical and underwriting expertise. In the report, the bar chart which showcased the degree of strength of the market characteristics indicated 'public awareness of the role of insurance' as the weakest characteristic which was followed closely by 'education and training'.

### **Reinsurance Sector Development in Nigeria**

Over the years, the reinsurance sector in Nigeria has grown rapidly. A key reason for this is the significant role of the National Insurance Commission (NAICOM) in reinsurance sector development. NAICOM was established in 1997 by the National Insurance Act of 1997. The Commission regulates and supervises the operations of insurance and reinsurance companies in Nigeria. Since the establishment of NAICOM, there have been several reforms in the reinsurance sector. Central to these reforms is the recapitalization of reinsurance companies which is meant to stabilize their capital structure. On May 20, 2019, NAICOM increased the minimum paid-up capital for reinsurance from N10 billion (USD 27.7million) to N20 billion (USD 55.5million). This move was taken to enable bigger and stronger reinsurance companies to emerge in the industry and to make liquid funds available for the prompt payment of claims. It is expected that the reforms will foster reinsurance sector development in Nigeria.

### **Reinsurance study in Nigeria**

In Nigerian universities, reinsurance study is embedded in the insurance degree programme (undergraduate/postgraduate). With respect to postgraduate degrees, a handful of universities offer insurance (and/or risk management) at the master's/Ph.D. level. These include; Gregory University, located in Abia state (South-East region); Enugu State University of Science and Technology, located in Enugu state ( South-East region); Imo State University, located in Imo state (South-East region); Joseph Ayo Babalola University, located at Osun state(South-West region); Lagos State University, located at Lagos state (South- West region); Niger Delta University, located at Bayelsa state (South-South region); Redeemer's University, located at Osun state (South-West region); the University of Lagos, located at Lagos (South-West region) and University of Uyo, located at Akwa Ibom state (South-South region).

A few scholars have conducted studies on reinsurance practice in Nigeria. Chukwudum (2019) conducted quantitative research on the reinsurance pricing of large motor insurance in Nigeria. The study employed negative binomial-generalized Pareto distribution to illustrate how extreme value theory can be used to model the tail of large losses from the Nigerian insurance sector. The results indicated that the GDP function serves as a good fit for the tails of large motor insurance claims. Abass (2019) focused on reinsurance dependence on the profitability of non-life insurance companies in Nigeria. The data were analyzed using regression techniques. The result of the study indicated that return on assets (ROA) and return on equity (ROE) have a significant impact on the profitability of non-life insurance companies in Nigeria. Abass and Ojikutu (2019) conducted an empirical study on the relationship between capital and demand for reinsurance in Nigeria. The study adopted a longitudinal descriptive research design. Data were analyzed using panel vector auto-regressive framework and granger causality test. Their results revealed that the

availability of capital increases the demand for reinsurance by non-life insurance companies in Nigeria. It was recommended that non-life insurance companies should take into consideration the availability of capital before underwriting risks. Aduloju and Ajemunigbohun (2017) studied the effect of reinsurance on the profitability of the ceding companies in Nigeria by adopting a descriptive research design. A correlation was also employed to analyze the data. The results indicated that a significant relationship exists between reinsurance capacity and gross written premiums of insurance companies in Nigeria. It was recommended that insurance companies should see reinsurance as an essential risk management mechanism. Obalola and Abass (2016) examined the demand for reinsurance and the solvency of insurance companies in Nigeria. The constant coefficient model was employed to analyze the data. Their findings suggest that there is a strong relationship between solvency and demand for insurance. They recommended that the cedant should focus more on the business mix, combined ratio and reinsurance pricing. Olajide et al. (2021) investigated hedging risk through a reinsurance mechanism. The study focused on insurance companies in Nigeria. Data were analyzed using regression analysis. Their results indicated that the reinsurance mechanism positively affects the financial performance of insurance companies in Nigeria. Soye et al. (2017) evaluated the effect of reinsurance on the sustainability of insurance firms in Nigeria. The study adopted an expo-facto research design. The dataset was analyzed using inferential statistics and linear regression analysis. The findings of the study indicated that the Net Retention ratio, Net commission ratio, Net Claim ratio and Ceded Reinsurance ratio have a positive impact on ROA and administrative expenses of insurance companies in Nigeria. An identical study that produced similar results was also carried out by Emmanuel (2021). Uche and Chikeleze (2001) investigated the origins and development of the Nigerian reinsurance sub-sector to assess the impact of the compulsory legal cession practice. They maintained that the practice was of no significance in today's world. It was therefore recommended that the government should concentrate on creating a favorable environment for the practice of insurance and reinsurance in Nigeria. Fagam and Akaaya (2015) reviewed the reinsurance legal framework in a bid to assess the strength and stability of Nigeria's reinsurance sub-sector with regard to the local protectionism and the domiciled-oriented regulatory approaches that its legal regime is currently based on. A quality-oriented approach was recommended.

The Nigerian literature landscape indicates a highly constrained coverage of reinsurance research works when compared to the different components that constitute the field. A higher proportion of quantitative studies cluster around portfolio analysis driven mainly by regression analysis. The aspects of reinsurance pricing, underwriting, claims analysis and risk management, in general, have been left almost bare. Thus, it is against the backdrop of education and training that this paper seeks to serve as a teaching aid to illustrate how reinsurance concepts and mathematical techniques can be applied with the help of the R programming language. The aim is to bridge the gap between theory and practice in reinsurance classes and to expose young researchers to a variety of techniques they can employ thereby strengthening their technical capacity.

The target audience includes master's and doctorate level students studying insurance in Nigerian institutions. Researchers, professionals and specialists with interest in the field of re/insurance within the country may find this tutorial useful as well.

The remaining part of the paper is subdivided as follows: section 2 briefly outlines the techniques employed and section 3 takes on the reinsurance R applications. Conclusions are drawn in section 4.

## METHODOLOGY

In pricing reinsurance, a couple of techniques can be applied. They include an excess of loss (XOL), quota share and stop loss reinsurance treaties. In-depth details of the different reinsurance treaties along with the analytical and numerical applications can be seen in the book written by Albrecher et al. (2017).

Let  $X_0$  represent the original loss which is divided between the cedant (also called the primary insurer, PI),  $X_{PI}$  and the reinsurer,  $X_{RI}$ . If the cedant's limit is  $D$  and the reinsurer's limit is  $E$ , then the XOL treaty is represented as  $E$  xs  $D$  read as  $E$  in excess of  $D$  where

$$X_{RI} = (X_0 - D)^+ - (X_0 - E - D)^+ \text{ and } X_{PI} = X_0 - X_{RI}$$

There are some features unique to XOL treaties, two of such are the annual aggregated limit (aal) and the annual aggregated deductible (aad). These features allow the reinsurer to limit the number of recoveries that the reinsured can collect in a given contract (in the case of aal) and/or reduce the cost of reinsurance for the cedant (in the case of aad). The latter also doubles as a self-protecting measure for the reinsurer. Here is an example that describes the process of how these features work.

For instance, let's say a ₦10 million x ₦10 million (which we simply write as, ₦10m x ₦10m) contract having ₦11m aad and ₦25m aal is taken up by an insurer. Furthermore, we assume that 3 losses of ₦20m each are observed in a given year. It, therefore, implies that the cedant will make recoveries as follows: For the first loss, no recoveries will be made because the value above the excess point of ₦10m is not greater than ₦11m (it is just exactly ₦10m). This is due to the aad value which only allows the reinsurer to start paying claims only when the recoverable amount exceeds ₦11m. Thus, in this case, the aad is exhausted. The second loss also has an excess of ₦10m, this makes the aggregate excess (from the first and second losses) ₦20m which is greater than the aad of ₦11m. This implies that the cedant will recover ₦9m (₦20m-₦11m). With respect to the third loss, the aggregate excess is now ₦30m which also exceeds the aad value. Here the cedant gets to recover the full value of ₦10m. This brings the total recovery amount to ₦19m which is less than the aal of ₦25m. Say, there is now the fourth loss of the same amount (₦20m), the cedant will only just be able to recover ₦6m from the reinsurer since ₦19m + ₦6m = ₦25m which exhausts the aal.

Mathematically, the recoveries having aal and aad features are computed using the formula

$$\text{Cedant's recoveries} = \min[\max\{\sum_{i=1}^n \min(\max(X_i - D, 0), E) - aad, 0\}, aal]$$

In the case of the quota share reinsurance, a fixed percentage,  $\alpha$  is used for sharing both the premium  $P_0$  and the loss,  $X_0$ . Therefore, the reinsurer's share of loss and premium becomes  $\alpha X_0$  and  $\alpha P_0$  respectively and that of the cedant is  $(1 - \alpha)X_0$  and  $(1 - \alpha)P_0$  respectively.

Stop loss reinsurance accounts for the aggregate loss ( $S$ ) over a given period, if we denote  $d$  as the deductible for the period then the stop-loss reinsurance payment will be:

$$S_{RI} = \begin{cases} 0 & \text{for } S \leq d \\ S - d & \text{for } S > d \end{cases}$$

We have only provided their general mathematical representations. Once a specific distribution is used to represent the loss, say in the XOL reinsurance, the formula is re-written in terms of the chosen distribution function. Kleiber and Kotz (2003) studied a variety of distribution models highlighting the similarities and differences as well as their characteristics. The ones employed in this study are:

### Poisson distribution

If a random variable  $X$  has the Poisson distribution with scale parameter  $\mu$ , then its probability mass function is denoted by

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}, \quad x = 0, 1, 2, \dots \quad (1)$$

### The extended Pareto distribution

This distribution which is widely used in property insurance due to its heavy-tailed characteristics is a generalization of the ordinary Pareto distribution. Its density and cumulative distribution functions are respectively,

$$f(x) = \frac{\alpha/\beta}{1+(\alpha/\beta)^{1+\alpha}} \quad \text{and} \quad F(x) = 1 - \frac{1}{(1+\alpha/\beta)^\alpha} \quad (2)$$

where the shape parameter is  $\alpha > 0$  and the scale parameter is  $\beta > 0$ . When an additional parameter ( $\theta$ ) is included, the density function takes the form

$$f(x) = \frac{\Gamma(\alpha+\theta)}{\Gamma(\alpha)\Gamma(\theta)} \frac{1}{\beta} \frac{x/\beta^{\theta-1}}{1+x/\beta^{\alpha+\theta}}, \quad \theta > 0$$

where  $\alpha, \beta, \theta > 0$ . When  $\theta$  is set to 1, we obtain the ordinary Pareto distribution and we can draw random variables using the inverse probability integral transform technique.

Thus, if  $U$  is a uniformly distributed variable on the interval  $[0, 1]$ . The inverse Pareto cumulative distribution function becomes

$$Z = \beta(U^{-1/\alpha} - 1) \quad (3)$$

### Lognormal distribution

$X \sim \text{lognormal}(\alpha, \beta)$  implies that the  $X$  is lognormally distributed with parameters  $\alpha$  and  $\beta$  and has a density function

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}} e^{-\frac{1}{2}(\ln(x/\alpha)/\beta)^2}, \quad x > 0, \alpha > 0, \beta > 0 \quad (4)$$

## Normal distribution

The normal distribution, also called Gaussian distribution, is a very common distribution function for independent, randomly generated variables. It takes the form of a bell-shaped curve and is characterized by two parameters - the mean,  $\mu$  and the standard deviation,  $\delta$ . The general form of its probability density function is

$$f(x) = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{1}{2}((x-\mu)/\delta)^2} \quad (5)$$

## REINSURANCE APPLICATIONS WITH R

The illustrative exercises in Bølviken (2014) serve as the general guide for this section which generally deals with simulated data. The other aspect of this section (3.4) makes use of the R package `reinsureR`, applied to a real dataset that comes from the Nigerian insurance sector. That is the cumulative inflation-adjusted paid claims development table for all claims (excluding large claims) extracted from the 2018 Allianz annual report. The accident years run from 2007 (the base year) to 2018 with 12 development lags. The incremental paid claims are extracted and used for this analysis. A basic knowledge of R is assumed. The discussions are embedded within each example as remarks.

### Reinsurance pricing

**Case 1:** Pricing the reinsurer's premium from an insurer's point of view.

Monte Carlo (MC) simulation technique is used to calculate the re-insurance premium for an excess of loss (XOL) contract (henceforth, termed a  $x$   $b$  contract) per risk/event.  $a$  is the cedant's retention and  $b$  is the reinsurer's limit.

#### Assumptions:

- ✓  $a=50$ ,  $b=500$  (in millions of naira).
- ✓ The claims are assumed to be Pareto distributed having shape ( $\alpha=2$ ) and scale ( $\beta=100$ ) parameters.
- ✓ The claim frequency  $\mu=0.01$  and the time  $T=1$  year. Hence the expected number of claims per annum is  $\mu*T=0.01$ .
- ✓  $Z$  represents the individual claims obtained from the inverse Pareto cumulative distribution function (cdf) described in section 2.
- ✓  $pi.re$  represents the pure premium of a reinsurer.
- ✓  $sd\_pi.re$  represents the standard error computed using Monte Carlo simulation of size  $m=100000$ .

#### **R-code**

```
MC.re=function(m=100000, muT=0.01, alpha=2, beta=100, a=50, b=500) {
```

```
Z=beta*(runif(m)**(-1/alpha)-1)
```

```
Z.re=pmin(pmax(Z-a,0), b) #reinsurer's portion.
```

```
pi.re=muT*mean(Z_re) #mean (Z_re) is the average excess losses
```

```
list(pi.re=pi.re, sd_pi.re=muT*sd(Z_re)/sqrt(m)) #it is divided by sqrt(m) to account for using a sample size of m.
```

```
}
```

```
MC.re()
```

## Results

```
pi.re= 0.512111; sd_pi.re=0.003354294
```

**Remark:** For the contract, the cedant is to pay a premium of N512,111 (+/- .34%) to the reinsurer. *pmin(excess loss, reinsurer's limit)* will allow us to choose the correct minimum value between the two stated options in the parenthesis.

**Case 2:** Computing (using MC) the total reinsurer's risk (overall loss) given several cedant portfolios, from the reinsurer's perspective.

Within this context, the case of a reinsurer who typically has access to the portfolios of multiple insurers is considered. The portfolio level or aggregate level losses from the several primary insurers' axb contracts is thus calculated and the total expected payout that will be made by the reinsurer (that is, the reinsurer's risk) is estimated.

### Assumptions:

- ✓ a=0.9, b=1.2 (in billions of naira).
- ✓ The total claims (X) are assumed to be normally distributed with a mean ( $\xi=1$ ) and standard deviation ( $\sigma=0.2$ ).
- ✓ G=20 represents the number of cedants' contracts.
- ✓ Y.re represents the total re-insurer risk.
- ✓ m=10000 represents the size of the Monte Carlo simulation.

### **R-code**

```
MC.re_risk=function(m, G, xi, sigma, a, b) {
```

```
X=rnorm(m*G, xi, sigma)# drawing normally distributed claims.
```

```
X=matrix(X, G, m)
```

```
X.re=pmin(pmax(X-a,0), b)
```

```
Y.re=apply(X.re, 2, sum)
```

```
list(Y.re=Y.re)
```

```
}
```

```
mean(MC.re_risk(m=10000, G=20, xi=1,sigma=0.2,a=0.9,b=1.2)$Y.re) #obtaining the expected reinsurer's risk
```

### **Result**



2.779166

**Remark:** The reinsurer will be expected to pay out a sum of approximately 2.78 billion nairas to all the cedants.

### Reinsurance reserves and solvency

**Case 3:** Calculating the required reinsurer's reserves based on the specified solvency criterion.

The numerical application of the *MC.re\_risk* function is carried out. The density function of the reinsurer's total risk is plotted and the solvency capital is obtained for varying levels of the cedant's retention (a) and the reinsurer's limit (b).

#### Assumptions:

- ✓ sol represents the reserves which is the solvency capital of the reinsurer
- ✓ epsilon represents the solvency criterion at the 95<sup>th</sup> and 99<sup>th</sup> percentiles.

#### **R-code**

```
Re.sol=function(m=10000,xi=1, sigma=0.2, a=0.9, b=1.2, G=20, epsilon=c(0.05,0.01)) {
```

```
Y.re=MC.re_risk(m, G, xi, sigma, a, b)$Y.re
```

```
d=density(Y.re, from=0)
```

```
plot(d$x, d$y, "l", xlab="Amount", ylab="")
```

```
title("Re-insurer expense")
```

```
sol=sort(Y.re)[m*(1-epsilon)] #sorts Y.re and extracts the solvency capital at 95% and 99% (1-epsilon reserve for  
#reinsurer)
```

```
Results=round(rbind(epsilon,sol),digits=2)
```

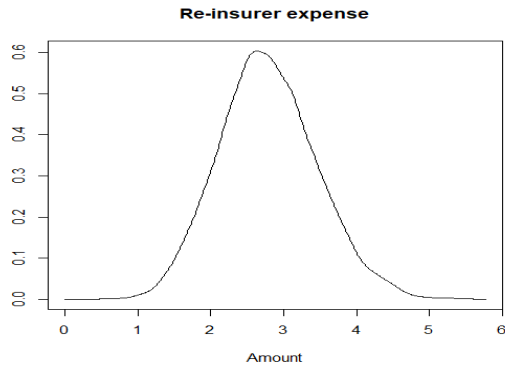
```
list(Results=Results)
```

```
}
```

```
Re.sol()
```

#### **Results**

When  $a=0.9$  and  $b=1.2$  for reinsurance contract  $a \times b$



**Figure 1:** Reinsurer’s total risk when  $a=0.9$  and  $b=1.2$

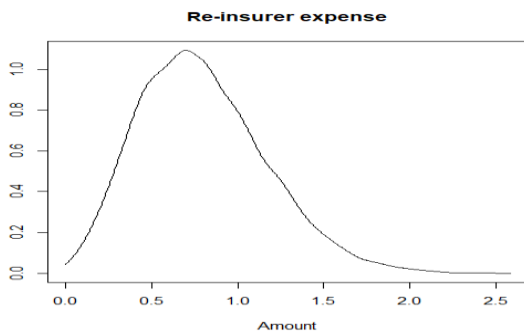
*Associated solvency capital at the 95<sup>th</sup> and 99<sup>th</sup> quantiles (that is, at the 5% and 1% upper percentiles)*

*epsilon 0.05 0.01*

*sol 3.92 4.45*

*When the limits change  $a=1.1$ ,  $b=1.4$  (that is, an increase in both cedant’s and reinsurer’s limits)*

*We notice a skewed distribution and the associated solvency reduces significantly*

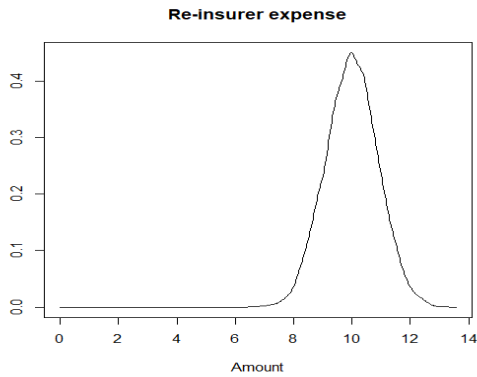


**Figure 2:** Reinsurer’s total risk when  $a=1.1$  and  $b=1.4$

*epsilon 0.05 0.01*

*sol 1.46 1.81*

*When  $a=0.5$  and  $b=1.4$  (which is a wider margin) we observe a left-skewed distribution (pronounced). This means that the probability of large losses will be higher for the reinsurer hence the need for higher reserves.*



**Figure 3:** Reinsurer’s total risk when  $a=0.5$  and  $b=1.4$

```
epsilon 0.05 0.01
sol      11.47 12.08
```

**Remark:** The reserves needed by the reinsurer is directly affected by the retention levels ( $a$  and  $b$ ) and the margin differences between them. For instance, a right-skewed distribution suggests a lower solvency value at the 5% and 1% upper percentile (Figure 2) while for a left-skewed distribution, the solvency value is high.

### Reinsurance pricing and portfolio analysis

**Case 4a:** A single portfolio with  $G$  identical policies having a  $x$   $b$  contract with fixed reinsurer’s limit.

This case focuses on a portfolio with losses all drawn from the same distribution. The pure reinsurance premium is also computed. Two sub-cases are considered. The first scenario (case 4a) allows the retention levels to vary but keeps the reinsurance limit constant and the second scenario (case 4b) allows for an unlimited reinsurance limit.

#### Assumption:

- ✓ Losses are assumed to be lognormal ( $Z$ ) with a mean ( $\theta=-0.5$ ) and standard deviation ( $\sigma=1$ )
- ✓  $G$  represents the number of policies,  $\mu$  the claim intensity, and  $T$  the time. Thus the expected number of claims is given by  $G\mu T$ .
- ✓  $pi.re$  represents the reinsurance premium.

#### **R-code (first scenario)**

```
Port.const_limit1=function(m, GmuT, theta, sigma, a, b){
  Z=rlnorm(m, theta, sigma)
  Z.re=pmin(pmax(Z-a,0), b)
  pi.re=round(GmuT*mean(Z.re), digits=2)
  list(pi.re=pi.re)}
```

Next, *Port.const\_limit1* will be implemented. The cedent's retention (lower limit) ,a, varies from 0 to 4. The reinsurer's limit, b, is fixed at 5. This is the upper bound.

```
Port.const_limit2=function(m=10000, GmuT=10, theta=-0.5, sigma=1, a=0:4, b=5){
  pi.re=a
  for(i in 1:length(a)) pi.re[i]=Port.const_limit1(m, GmuT, theta, sigma, a[i], b)$pi.re
  Results=rbind(a, pi.re)
  list(Results=Results)}
```

### Results

```
a    0.00  1.00  2.00  3.00  4.00
pi.re 9.56  3.41  1.47  0.97  0.57
```

**Remark:** The reinsurance premium reduces as the retention level increases.

**Case 4b:** A single portfolio with G identical policies having a x b contract with infinite reinsurer's limit.

Here, the cedent's responsibility with respect to the reinsurance contract is of interest. That is, the net claim payments that the cedent makes are simulated *m* times. A frequency-severity approach is applied where both the claim numbers and the claim sizes are modeled.

### Assumption:

- ✓ The claim numbers are assumed to be Poisson-distributed and the claim sizes are still lognormal.
- ✓ *X.ce* represents the cedent's portion.

### **R-code (second scenario)**

```
Port.unlimited1=function(m,JmuT,theta,sigma,a) {
  N=rpois(m,JmuT) #generating Poisson-distributed claim numbers
  X.ce=array(0,m)
  for (i in 1:m){
    Z=rlnorm(N[i],theta,sigma)
    X_ce[i]=sum(pmin(Z,a)) #cedent's portion
  }
  list(X.ce=X.ce)}
```

*Port.unlimited1* is now implemented. The cedent's retention (lower limit),  $a$ , is varied from 1 to 3. The reinsurer's limit,  $b$ , is unlimited. A further examination of the cedent's solvency based on the 0.05 and 0.01 upper percentiles (solvency levels) is carried out. The cedent's net payout density function is also plotted.

**R-code**

```
Port.unlimited2=function(m=10000, GmuT=10, theta=-0.5, sigma=1,a=1, epsilon=c(0.05,0.01)){
X.ce=Port.unlimited1(m, GmuT, theta, sigma, a)$X.ce

d=density(X.ce)

plot(d$x, d$y,"l", xlab="Amount", ylab=" ")

title("Cedent responsibility")

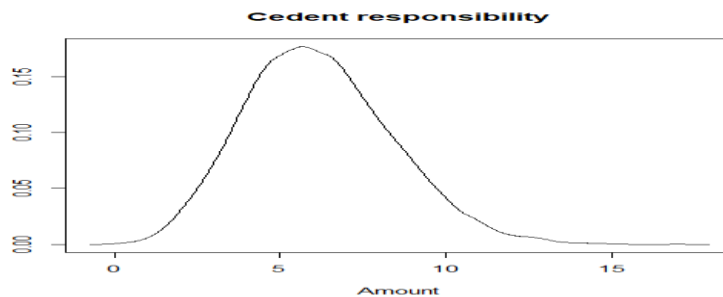
sol=sort(X.ce)[(1-epsilon)*m] #solvency of cedent

Results=round(rbind(epsilon, sol), digits=2)

list(Results=Results)}
```

**Results**

When  $a=1$



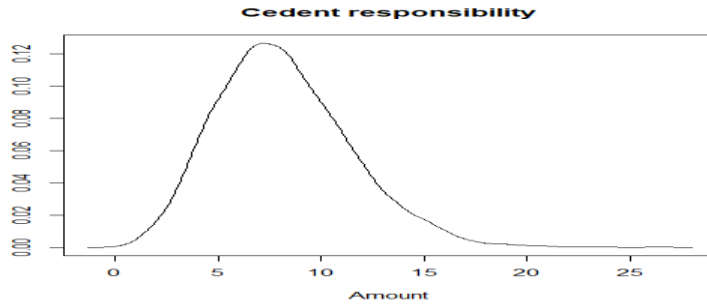
**Figure 4:** Cedent's responsibility for an  $a \times b$  reinsurance contract

where the retention,  $a=1$  with no upper bound.

*epsilon* 0.05 0.01

*sol* 10.04 11.94

When  $a=2$



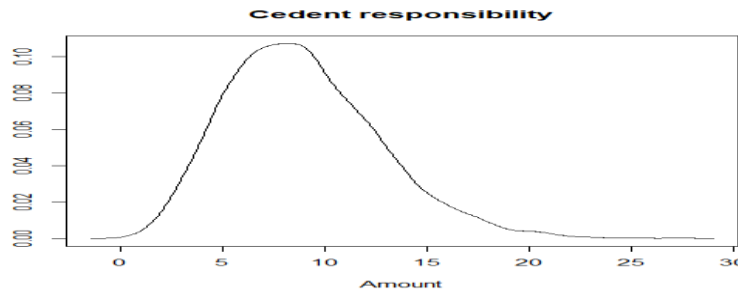
**Figure 5:** Cedent's responsibility for an  $a \times b$  reinsurance contract

where the retention,  $a=2$  with no upper bound.

*epsilon 0.05 0.01*

*sol 13.99 16.81*

When  $a=3$



**Figure 6:** Cedent's responsibility for an  $a \times b$  reinsurance contract

where the retention,  $a=3$  with no upper bound.

*epsilon 0.05 0.01*

*sol 15.68 19.21*

**Remark:** When the retention level goes down, the insurer's capital requirements (solvency) go down as well. This is because he passes more risk to the reinsurer. Hence, his reinsurance premium is higher.

### Comparing different reinsurance program designs

**Case 5:** Incremental claims of  $n = 12$  years (2007-2018) are extracted from the cumulative run-off triangle of the Allianz Nigeria annual report of 2018 (<https://www.allianz.ng/About-Us/Annual-report.html>). This is considered one portfolio. The application of different treaty terms such as the annual aggregated deductible (aad), the annual aggregated limit (aal), reinstatements (rns) and deductibles (ded) are included to compute the final reinsurance recoveries. All claims are in millions of naira.

**R-code**

`n=12`

```
A.Claims <-data.frame(origin=factor(rep(2007:2018, n:1)), dev=sequence(n:1), incremental.paid=c(109278, 227440, 21224, 20672, 3032, 2252, 212, 6, 6, 6, 6, 6, 119908, 113964, 31403, 18735, 7667, 3534, 3753, 3753, 3753, 628, 65841, 97110, 19983, 11027, 6507, 3073, 335, 13, 221, 221, 90728, 102345, 20328, 7884, 4118, 569, 326, 326, 320, 120225, 87624, 25847, 16905, 2928, 4255, 344, 375, 103577, 100933, 26000, 2948, 2167, 326, 2066, 141682, 80692, 12623, 941, 1141, 950, 147677, 96978, 21480, 6096, 2291, 46817, 39388, 9263, 6218, 67437, 57073, 11699, 48015, 49050, 48415))  
  
>head(A.Claims, 15) #this gives the first 15 entries of A.Claims (Table 1).
```

**Table 1:** First 15 entries of the Allianz (2018) incremental claims.

Origin	Dev	Incremental.paid
2007	1	109278
2007	2	227440
2007	3	21224
2007	4	20672
2007	5	3032
2007	6	2252
2007	7	212
2007	8	6
2007	9	6
2007	10	6
2007	11	6
2007	12	6
2008	1	119908
2008	2	113964
2008	3	31403

dev refers to the development years.

```
#Adding all entries in each year and renaming the columns of A.Claims_agg to align with that of A.Claims  
A.Claims_agg <- aggregate(incremental.paid ~ origin, A.Claims, sum)  
colnames(A.Claims_agg)=c("year","amount")  
  
#Creating the premium (p) table. Here, the average yearly claims are taken as the claims.  
p <- aggregate(Incremental.paid ~ origin, Claims, mean)  
p$Incremental.paid <- mean(p$Incremental.paid)  
colnames(p)=c("year","amount")  
  
#Making use of the claims function from the reinsureR package which defines an object of class Claims
```

```
library(reinsureR)
```

```
claims1 <- claims( A.Claims_agg, p )#defining an object of class Claims, showing the table of claims and premiums.
```

```
> claims1
```

**Table 2:** Dataframe showing Claims object class

Year	Portfolio	Amount.clm	Amount.prm
2007	0	384140	36215.26
2008	0	310851	36215.26
2009	0	204331	36215.26
2010	0	226944	36215.26
2011	0	258503	36215.26
2012	0	238017	36215.26
2013	0	238029	36215.26
2014	0	274522	36215.26
2015	0	101686	36215.26
2016	0	136209	36215.26
2017	0	97065	36215.26
2018	0	48415	36215.26

*portfolio: this refers to the portfolio associated with the considered claim.*

```
#Defining the various insurance treaties.
```

```
#Excess of loss.
```

```
treaty_1 <- xl(ded = 100000, lim = 20000, aad = 5000, aal = 200000, prm = 0.01, rns = 1) #prm is the premium rate
```

```
claims_treaty1 <- apply_treaty(claims1, treaty_1)
```

```
#quota share treaty
```

```
claims1 <- claims( A.Claims_agg, p )#resetting to the original dataset of claims and premiums.
```

```
treaty_2 <- qs(0.8, com = 0.25) #com refers to commission
```

```
claims_treaty2 <- apply_treaty(claims1, treaty_2)
```

```
#stop loss treaty
```

```
claims1 <- claims( A.Claims_agg, p )
```

```
treaty_3 <- sl(ded = 100000, lim = 20000, prm = 0.01, ptf = "all") # ptf provides a list of portfolios on which the
```

```
#treaty is to be applied on
```

```
claims_treaty3 <- apply_treaty(claims1, treaty_3)
```



#printing out the summary and tables

A.Claims\_agg

summy(claims\_treaty1)

summy(claims\_treaty2)

summy(claims\_treaty3)

### Results

>A.Claims\_agg

**Table 3:** Aggregated claims across the years.

Year	Amount
2007	384140
2008	310851
2009	204331
2010	226944
2011	258503
2012	238017
2013	238029
2014	274522
2015	101686
2016	136209
2017	97065
2018	48415

>summy(claims\_treaty1)

**Table 4:** Excess of loss treaty showing the respective values when the different treaty terms are applied.

Year	amount.c lm	amount_after_ treaty_1.clm	amount. prm	amount after treaty 1. prm	Reinstatement	Comm- ission	rns gain
2007	384140	369140	36215.26	35853.11	271.6145	0	14366.2329
2008	310851	295851	36215.26	35853.11	271.6145	0	14366.2329
2009	204331	189331	36215.26	35853.11	271.6145	0	14366.2329
2010	226944	211944	36215.26	35853.11	271.6145	0	14366.2329
2011	258503	243503	36215.26	35853.11	271.6145	0	14366.2329
2012	238017	223017	36215.26	35853.11	271.6145	0	14366.2329
2013	238029	223029	36215.26	35853.11	271.6145	0	14366.2329
2014	274522	259522	36215.26	35853.11	271.6145	0	14366.2329
2015	101686	101686	36215.26	35853.11	0.000	0	-362.1526
2016	136209	121209	36215.26	35853.11	271.6145	0	14366.2329
2017	97065	97065	36215.26	35853.11	0.000	0	-362.1526
2018	48415	48415	36215.26	35853.11	0.000	0	-362.1526

clm refers to claim

> summy(claims\_treaty2)

**Table 5:** Quota share treaty showing the respective values when the different treaty terms are applied.

Year	Amount. Clm	Amount after treaty 2. Clm	Amount. Prm	Amount after treaty 2. Prm	Reinstatement	Commission	Rns gain
2007	384140	76828.0	36215.26	7243.053	0	7243.053	285582.84
2008	310851	62170.2	36215.26	7243.053	0	7243.053	226951.64
2009	204331	40866.2	36215.26	7243.053	0	7243.053	141735.64
2010	226944	45388.8	36215.26	7243.053	0	7243.053	159826.04
2011	258503	51700.6	36215.26	7243.053	0	7243.053	185073.24
2012	238017	47603.4	36215.26	7243.053	0	7243.053	168684.44
2013	238029	47605.8	36215.26	7243.053	0	7243.053	168694.04
2014	274522	54904.4	36215.26	7243.053	0	7243.053	197888.44
2015	101686	20337.2	36215.26	7243.053	0	7243.053	59619.64
2016	136209	27241.8	36215.26	7243.053	0	7243.053	87238.04
2017	97065	19413.0	36215.26	7243.053	0	7243.053	55922.84
2018	48415	9683.0	36215.26	7243.053	0	7243.053	17002.84

> *summy(claims\_treaty3)*

**Table 6:** Stop loss treaty showing the respective values when the different treaty terms are applied.

Year	Amount . Clm	Amount after treaty 3. Clm	Amount. Prm	Amount after treaty 3.prm	Reinstatement	Commission	Rns gain
2007	384140	364140	36215.26	35853.11	0	0	19637.8474
2008	310851	290851	36215.26	35853.11	0	0	19637.8474
2009	204331	184331	36215.26	35853.11	0	0	19637.8474
2010	226944	206944	36215.26	35853.11	0	0	19637.8474
2011	258503	238503	36215.26	35853.11	0	0	19637.8474
2012	238017	218017	36215.26	35853.11	0	0	19637.8474
2013	238029	218029	36215.26	35853.11	0	0	19637.8474
2014	274522	254522	36215.26	35853.11	0	0	19637.8474
2015	101686	100000	36215.26	35853.11	0	0	1323.8474
2016	136209	116209	36215.26	35853.11	0	0	19637.8474
2017	97065	97065	36215.26	35853.11	0	0	-362.1526
2018	48415	48415	36215.26	35853.11	0	0	-362.1526

**Remark:** In the case of the XOL, referring to Table 4 column 3 (*amount\_after\_treaty\_1.clm*), only ₦15,000,000 (₦15000m) is paid out once the following two conditions are satisfied - the first being that the claim amount is above ₦100000m and the aad is greater than ₦5000m (that is,  $P(aad > ₦5000m | claim > ₦100000m)$ ). Thus we see that although the claims are greater than ₦100000m in the year 2015, the aad is less than ₦5000m, hence nothing is paid. In column 5 (*amount\_after\_treaty\_1.prm*) only 1% ( ₦362.1526m) of the premium is paid to the reinsurer, leaving ₦35853.11m for the insurer. The rns gain column gives the value of the cedant's gain from reinsurance. This value is negative in the instances where no claims are paid (2015, 2017 and 2018), indicating that the money that the cedant loses is the premium paid out to the reinsurer. In comparison, with respect to the stop loss treaty, there is no aad. Thus it can be noted that ₦1686m is paid out leaving the deductible amount of ₦100000m (Table 6, column 3: *amount after treaty 3. clm*, the year 2015) which gives rise to a positive rns gain.

In general, this follows a simple cost-benefit analysis and does not account for the stability of the insurer's surplus or consider the freed-up risk capital that reinsurance offers.

## CONCLUSION

Given the lack of technical capacity in teaching the reinsurance course in Nigeria, this study attempts to bridge this gap by providing a teaching aid to strengthen the technical capacity of researchers and users in this field. The tutorial involved the application of some quantitative concepts in reinsurance which was illustrated using the R programming language. The study dwelt on reinsurance pricing both from the primary insurer's point of view and the reinsurer's perspective. Some treaties considered include the excess of loss, quota share and stop loss policies in a given portfolio. The impact of reinsurance on the insurer's capital requirement, reserves and solvency was also assessed.

## AUTHOR DECLARATIONS

**Funding:** No funding was received for conducting this research.

**Conflicts of interest/Competing interests:** The authors have no conflicts of interest to declare.

**Availability of data and material/Data availability:** The datasets generated and/or analyzed during the current study are mainly simulated. The real dataset used is freely available online at <https://www.allianz.ng/About-Us/Annual-report.html>

**Code availability:** The codes are embedded in the study. Additionally, the reinsureR R package was used.

## AUTHORS' CONTRIBUTIONS

**Queensley C. Chukwudum:** Conceptualization; data curation; methodology; writing; modeling and interpretation of data. **Mfonabasi Ekanem:** Literature review; writing.

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